CBCS SCHEME

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18MAT31

Third Semester B.E. Degree Examination, Jan./Feb. 2021
Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1 a. Find the Laplace transform of $\cos t \ \cos 2t \ \cos 3t$. **(06 Marks)**

b. If $f(t) = \begin{cases} t, & 0 < t < a \\ 2a - t, & a < t < 2a \end{cases}$ and $f(t + 2a) = f(t)$, show that $L\{f(t)\} = \frac{1}{s^2} \tanh\left(\frac{as}{2}\right)$. **(07 Marks)**

c. Find the Inverse Laplace transforms of :

i) $\dfrac{2s+1}{s^2+6s+13}$ ii) $\dfrac{1}{3}\log\left(\dfrac{s^2+b^2}{s^2+a^2}\right)$. **(07 Marks)**

OR

2 a. Express the function f(t) in terms of unit step function and find its Laplace transform, where

$$f(t) = \begin{cases} 1, & 0 < t \le 1 \\ t, & 1 < t \le 2 \\ t^2, & t > 2 \end{cases}$$ **(06 Marks)**

b. Find the Inverse Laplace transform of $\dfrac{s^2}{(s^2+a^2)^2}$ using Convolution theorem. **(07 Marks)**

c. Solve by the method of Laplace transforms, the equation
$y'' + 4y' + 3y = e^{-t}$ given $y(0) = 0$, $y'(0) = 0$. **(07 Marks)**

Module-2

3 a. Obtain the Fourier series of the function $f(x) = x^2$ in $-\pi \le x \le \pi$. **(06 Marks)**

b. Obtain the Fourier series expansion of

$$f(x) = \begin{cases} x, & 0 < x < \pi \\ x - 2\pi, & \pi < x < 2\pi \end{cases}$$ **(07 Marks)**

c. Find the Cosine half range series for $f(x) = x(\ell - x)$, $0 \le x \le \ell$. **(07 Marks)**

OR

4 a. Obtain the Fourier series of $f(x) = |x|$ in $(-\ell, \ell)$. **(06 Marks)**

b. Find the sine half range series for

$$f(x) = \begin{cases} x, & 0 < x < \pi/2 \\ \pi - \pi, & \pi/2 < x < \pi \end{cases}$$ **(07 Marks)**

c. Obtain the constant term and the coefficients of the first cosine and sine terms in the Fourier expansion of y from the table. **(07 Marks)**

x	0	1	2	3	4	5
y	9	18	24	28	26	20

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-3

5 a. If $f(x) = \begin{cases} 1-x^2 & , |x|<1 \\ 0 & , |x| \geq 1 \end{cases}$. Find the Fourier transform of f(x) and hence find value of $\int_0^\infty \frac{x\cos x - \sin x}{x^3}dx$. **(06 Marks)**

b. Find the Fourier Cosine transform of
$$f(x) = \begin{cases} 4x, & 0<x<1 \\ 4-x, & 1<x<4 \\ 0, & x>4 \end{cases}$$ **(07 Marks)**

c. Find the Z – transform of $\cos\left(\frac{n\pi}{2}+\frac{\pi}{4}\right)$. **(07 Marks)**

OR

6 a. Solve the Integral equation
$\int_0^\infty f(\theta)\cos\alpha\theta\, d\theta = \begin{cases} 1-\alpha & , 0 \leq \alpha \leq 1 \\ 0 & , \alpha > 1 \end{cases}$ hence evaluate $\int_0^\infty \frac{\sin^2 t}{t^2}dt$. **(06 Marks)**

b. Find the Inverse Z – transform of $\frac{2z^2+3z}{(z+2)(z-4)}$. **(07 Marks)**

c. Using the Z – transform, solve $Y_{n+2} - 4Y_n = 0$, given $Y_0 = 0$, $Y_1 = 2$. **(07 Marks)**

Module-4

7 a. Using Taylor's series method, solve the Initial value problem
$\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$ at the point $x = 0.1$. Consider upto 4^{th} degree term. **(06 Marks)**

b. Use modified Euler's method to compute y(0.1), given that $\frac{dy}{dx} = x^2 + y$, $y(0) = 1$ by taking $h = 0.05$. Consider two approximations in each step. **(07 Marks)**

c. Given that $\frac{dy}{dx} = x - y^2$, find y at x = 0.8 with

x :	0	0.2	0.4	0.6
y :	0	0.02	0.0795	0.1762

By applying Milne's method. Apply corrector formula once. **(07 Marks)**

OR

8 a. Solve the following by Modified Euler's method
$\frac{dy}{dx} = x + |\sqrt{y}|$, $y(0) = 1$ at $x = 0.4$ by taking $h = 0.2$. Consider two modifications in each step. **(06 Marks)**

b. Given $\frac{dy}{dx} = 3x + \frac{y}{2}$, $y(0) = 1$. Compute y(0.2) by taking $h = 0.2$ using Runge – Kutta method of order IV. **(07 Marks)**

c. Given $\frac{dy}{dx} = (1+y)x^2$ and $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.979$, determine y(1.4) by Adam's Bashforth method. Apply corrector formula once. **(07 Marks)**

Module-5

9 a. Given $y'' - xy' - y = 0$ with $y(0) = 1$, $y'(0) = 0$. Compute $y(0.2)$ using Runge – Kutta method. **(06 Marks)**

b. Derive Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. **(07 Marks)**

c. Prove that the geodesics on a plane are straight lines. **(07 Marks)**

OR

10 a. Find the curve on which functional $\int_0^1 [(y')^2 + 12xy]dx$ with $y(0) = 0$, $y(1) = 1$ can be extremized. **(06 Marks)**

b. Obtain the solution of the equation $\frac{2d^2y}{dx^2} = 4x + \frac{dy}{dx}$ by computing the value of dependent variable corresponding to the value 1.4 of the independent variable by applying Milne's method using the following data. Apply corrector formula once. **(07 Marks)**

x :	1	1.1	1.2	1.3
y :	2	2.2156	2.4649	2.7514
y' :	2	2.3178	2.6725	3.0657

c. A heavy cable hangs freely under gravity between two fixed points. Show that the shape of the cable is Catenary $y = c\cosh\left(\frac{x+a}{c}\right)$. **(07 Marks)**

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